12 HYDRAULIC FUNDAMENTALS

12.1 INTRODUCTION

The planning and design of stormwater management and drainage system facilities require understanding and application of fundamental hydraulics of water movement as well as pollutant transport processes.

This chapter briefly discusses concepts, principles and formulae that are inherent in more modern urban stormwater system including for open channels, pipes, ponds, porous media and other structures.

As most flows in stormwater system are practically in unsteady-nonuniform conditions during periods of rainfall-runoff events, the chapter in particular covers these hydraulic principles for use in performing flow routing in Chapter 14 and pollutant transport in Chapter 15 and subsequently applied in the design of the system.

In a simple/isolated stormwater system, such as small conveyances and on-site facilities (minor system), where transient flow conditions can be ignored the lumped peak-steady uniform flow formulae are used and they are adequately covered in this chapter.

12.2 GENERAL PRINCIPLES

12.2.1 Basic Definitions

(a) Fluid Properties

The *density* (ρ) of a fluid (Roberson and Crowe, 1993) is its mass per unit volume, while the *specific weight* (γ) is its weight per unit volume. The density and specific weight are related by the equation:

$$\gamma = \rho g \tag{12.1}$$

in which g = acceleration due to gravity. In SI units, ρ is expressed in kg/m^3 and for specific weight is N/m^3.

An *ideal fluid* may be defined as one in which there is no friction, i.e., viscosity is zero. In a *real fluid*, shear force exists whenever motion takes place, thus producing fluid friction. An ideal fluid does not exist in reality, but the concept is useful in simplifying many analyses. The *viscosity* of a fluid is a measure of its resistance to shear or angular deformation. If *du/dy* is the velocity gradient and τ is the shearing stress between any two thin sheets of fluid, then:

$$\tau = \mu \frac{du}{dy} \tag{12.2}$$

The coefficient μ is called the *absolute* or *dynamic viscosity* and its units are poise. The *kinematic viscosity* ν is defined as the dynamic viscosity μ divided by the mass density ρ, i.e.:

$$\nu = \frac{\mu}{\rho} \tag{12.3}$$

and is expressed in stokes.

(b) Fluid Flow

Velocity is the linear rate of movement or displacement of a point with respect to time (in meter per second, m/s). *Discharge* is the quantity/volume of liquid flowing past a given point/section per unit time (in cubic meters per second, m^3/s). If the flow velocity (v) varies across the section, then flow:

$$Q = \int_A v dA = VA \tag{12.4}$$

where

v = velocity through infinitesimal area dA

V = mean velocity over the section

(i) Steady and Unsteady Flow

Steady flow exists if the velocity at a point remains constant with respect to time. Conversely, unsteady flow exists if the velocity changes either in magnitude or in direction with respect to time. Steady flow is usually much easier to analyse and solve than unsteady flow. In fact a strictly rigorous solution for unsteady flow is sometimes impossible, so that only approximate solutions are possible.

(ii) Uniform and Nonuniform Flow

If at a given instant the velocity remains constant with respect to distance along a streamline, the flow is uniform. If there is a change either in magnitude or in direction along the streamline, the flow is nonuniform. In flow around a bend of a pipe or channel, the direction changes with distance and in flow with changing cross section, the magnitude changes with distance, hence the flow is also nonuniform.

12.2.2 Governing Equation of Motion

Hydraulically the flows in stormwater system, through conveyance, detention or retention facilities are solved by application of the principles of continuity, momentum, and energy. This section presents the general principles, which are later used in developing specific flow equations through conveyance, pond and porous media.

(a) Continuity Equation

The differential form of the continuity equation in three-dimensional fluid space, for either steady or unsteady, is derived based on control volume shown in Figure 12.1, with sides having length dx, dy and dz. Let the velocity components in *x*, *y* and *z* direction be *u*, *v* and *w*, respectively.

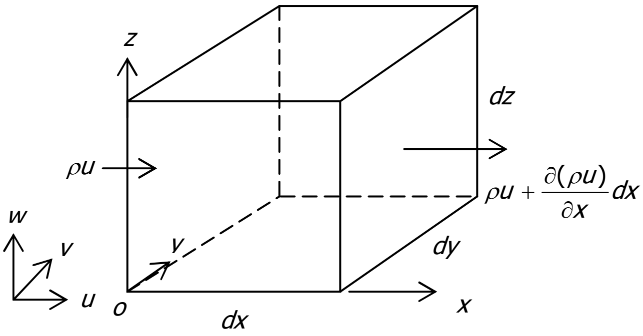


Figure 12.1 Flow Control Volume

According to Newtonian physics, mass must be conserved, i.e. the sum of the rates of mass inflow to the control volume is equal to the time rate of change of the mass in the control volume.

In x-direction for example mass flow rate is:

$$\rho u\,dy\,dz - \left\{\rho u + \left[\frac{\partial(\rho u)}{\partial x}\right]dx\right\}dy\,dz = \frac{\partial\rho}{\partial t}dxdydz \qquad (12.5)$$

$$-\frac{\partial\rho\mu}{\partial x}dxdydz = \frac{\partial\rho}{\partial t}dxdydz \qquad (12.6)$$

For the control volume, in three dimensions, the equation is reduced to:

$$-\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z} = \frac{\partial\rho}{\partial t} \qquad (12.7)$$

which is the equation of continuity in its most general form. For steady incompressible fluid flow, it forms:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (12.8)$$

For steady flow from section to section and average velocity is used for each section, the continuity equation can be written as:

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3 \qquad (12.9)$$

in which the subscripts refer to different section. The equation is valid if there is no inflow or outflow between the sections.

(b) Momentum Equation

By Newton's second law of motion, the time rate of change of momentum is equal to the net force applied in a given direction (Figure 12.2). A momentum equation for an unsteady, nonuniform flow is:

$$\sum F = \frac{d}{dt}\int_{\forall} V\rho \bullet d\forall + \int_S V\rho V \bullet dA \qquad (12.10)$$

If nonuniform flow is steady it forms:

$$\sum F = \int_S V\rho V \bullet dA \qquad (12.11)$$

For a steady uniform flow it results in the forces applied to the system are in equilibrium:

$$\sum F = 0 \qquad (12.12)$$

There are three forces acting on the fluid control volume: friction, gravity and pressure.

The momentum equation may be written for a volume of water between two cross sections in one-dimensional flow as:

$$\sum F = \rho Q\left(V_{out} - V_{in}\right) \qquad (12.13)$$

in which

ΣF = vectorial sum of the component of all the external forces acting on the water in the flow direction

V_{out} = flow velocity at the downstream cross section

V_{in} = flow velocity at the upstream cross section

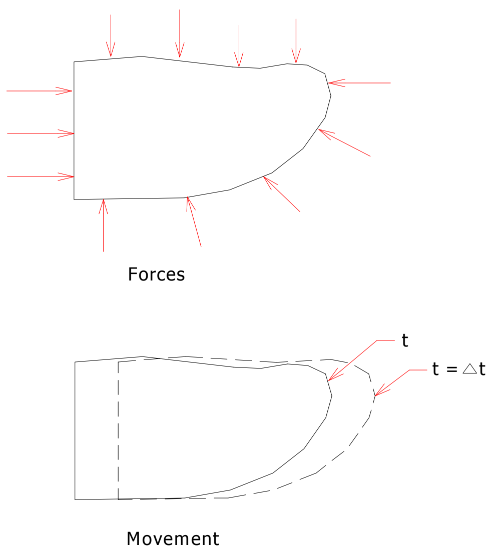


Figure 12.2 Fluid Mass Control Volume

(c) Energy Equation

The law of conservation of energy is well known. It applies to steady state fluid flows. The energy equation can be derived from Newton's second law of motion. For one-dimensional, or irrotational, steady flow, the following energy equation can be written for any two cross sections 1 and 2 of the flow:

$$K_{e1}\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1+E_m+E_H=K_{e2}\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2+h_L \quad (12.14)$$

where K_{e1} $(V_1^2/2g)$ and K_{e2} $(V_2^2/2g)$ are velocity heads at sections 1 and 2, respectively; p_1/γ and p_2/γ are the pressure heads at the two sections; z_1 and z_2 are elevation heads, or the elevation of the two sections above a certain datum plane; E_M is the mechanical energy added between the sections; E_H is the heat energy added between the sections; h_L is the head loss; K_{e1} and K_{e2} are energy-flux correction factor for the two sections; V_1 and V_2 are average velocities at the two sections; p_1 and p_2 are pressures at the two sections; g is the gravitational acceleration; and γ is the specific weight. The sum of the pressure head and the elevation head is termed the piezometric head, $h = (p/\gamma) + z$.

The velocity distribution across the flow is usually nonuniform. Since the average velocities are used the effect of nonuniform distribution of velocity is corrected by K_e, which is defined as:

$$K_e=\frac{1}{A}\int_A\left(\frac{v}{V}\right)^3 dA \quad (12.15)$$

where A is the cross-sectional area of flow, v is the local velocity for the incremental area dA, and V is the average velocity over the area A. In practice, K_e is usually taken as 1.0.

The energy equation, which contains scalar quantities, can be applied to the solution of such problems as jets issuing from an orifice, flow under a gate, flow over a weir, siphons, transition flow in pipes and open channels, flow associated with pumps and flow through porous media. Furthermore, such phenomena frequently exist in flow systems and can sometimes be used as a means of measuring velocity, pressure, or discharge of the flow.

The term $z + (p/\gamma)$ represents the level to which liquid will rise in a piezometer tube. The piezometric head line, or hydraulic grade line (HGL), is a line drawn through the tops of the piezometer columns. A pitot tube, a small open tube with its open end pointing upstream, will intercept the kinetic energy of the flow and hence indicate the total energy head, $z + p/\gamma + V^2/2g$. Referring to Figure 12.3, which depicts the flow of an ideal fluid, the vertical distance from point A on the stream tube to the level of the piezometric head at that point represents the pressure head in the flow at point A. The vertical distance from the liquid level in the piezometer tube to that in the pitot tube is $V^2/2g$. In Figure 12.3 the horizontal line sketched through the pitot-tube liquid levels is known as the energy grade line (EGL). For flow of an ideal fluid, the energy line is horizontal since there is no head loss.

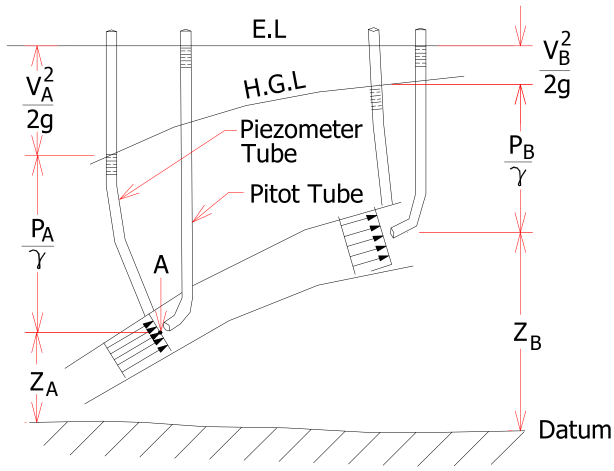


Figure 12.3 Conservation of Energy in Ideal Fluid

(Daugherty and Franzini, 1977)

12.3 STEADY OPEN CHANNEL FLOW

Open channels include not only those, which are completely, open overhead, but also closed conduits which are flowing partly full. Examples of such closed conduits are tunnels, storm pipes, culverts and various types of pipelines.

Flow in open channels involves a free surface, which is actually an interface between two fluids having different specific weights, such as air and water. Steady flow in open channels has three classifications.

1. Uniform or nonuniform,
2. Laminar or turbulent, and
3. Tranquil (subcritical), rapid (supercritical), or critical.

Strictly uniform flow rarely exists. For practical purposes, flow in an open channel is generally considered as uniform if the depth of flow is approximately constant in the direction of flow. The depth of uniform flow is called *normal depth* (y_0). The nonuniform flow is divided into gradually and rapidly varied flows (Figure 12.4).

Whether laminar flow or turbulent flow exists in an open channel depends upon the Reynolds number (Re) of the flow, just as it does in pipes. Like the flow in pipes, turbulent flow may be over either a smooth boundary or a rough boundary, depending on the relative size of the roughness elements as compared with the thickness of the laminar sublayer.

Unlike laminar and turbulent flow, tranquil flow and rapid flow occur only with a free surface or interface. The criterion for this classification of flow is the Froude number $Fr = V/\sqrt{(gy)}$. When Fr = 1.0, the flow is critical; when $Fr < 1$, the flow is tranquil; and when $Fr > 1$, the flow is rapid.

Uniform flow in an open channel occurs with either a mild, a critical, or a steep slope, depending on whether the flow is tranquil, critical, or rapid, respectively.

12.3.1 Uniform Flow Formula

Two most common equations for uniform flow in open channels are the Chezy and the Manning equations.

(a) The Chezy Formula

This equation, proposed by Chezy in 1769, may be written as:

$$V = C\sqrt{RS} \qquad (12.16)$$

where V is the mean velocity of flow, C is the Chezy discharge coefficient, R is the hydraulic radius and S is the slope of the channel or the sine of the slope angle.

For laminar flow in a wide channel, assuming a parabolic distribution of velocity the value of C can be determined by the following equation:

$$\frac{C}{\sqrt{g}} = \sqrt{\frac{\mathrm{Re}}{8}} \qquad (12.17)$$

where,

Re = $4VR/\nu$

For turbulent flow in a wide channel, the velocity distribution may be assumed to be logarithmic, as:

$$\frac{(v - V)\,C}{V\sqrt{8g}} = 2\log\frac{y}{y_0} + 0.88 \qquad (12.18)$$

where v is the local velocity at a depth y, and y_0 is the total depth. This equation, however, does not apply near the bed or near the surface of the flow.

In alluvial channels the magnitude of C depends upon the form of the boundary roughness.

Expressed in terms of the Darcy-Weisbach resistance coefficient f, the coefficient C is:

$$C = \sqrt{\frac{8g}{f}} \qquad (12.19)$$

(b) The Manning Formula

In an effort to correlate and systematise existing data from natural and artificial channels, Manning in 1889 proposed an equation which was developed into:

$$V = \frac{1}{n}R^{2/3}S^{1/2} \qquad (12.20)$$

where n is the Manning roughness coefficient. By comparing this equation with the Chezy equation the following relationship can be written:

$$C = \frac{R^{1/6}}{n} \qquad (12.21)$$

This relationship indicates that the Chezy discharge coefficient is a function of the Manning coefficient and the hydraulic radius. The Manning n was developed empirically as a coefficient which remained approximately a constant for a given boundary condition, regardless of slope of channel, size of channel, or depth of flow. As a matter of fact, however, each of these factors causes n to vary to some extent. In other words, the Reynolds number, the shape of the channel, and the relative roughness have an influence on the magnitude of Manning's n.

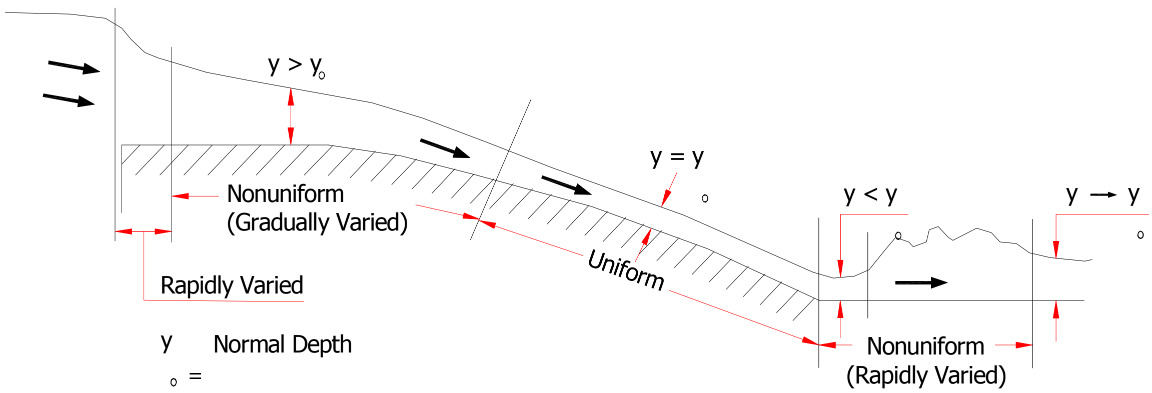


Figure 12.4 Types of Steady Flows in Channel

(c) Specific-head Diagram

The following specific head or specific energy equation is a very useful tool in analysing the flow in open channels.

$$H = y + \frac{V^2}{2g} \tag{12.22}$$

For a rectangular channel:

$$H = y + \frac{q^2}{2gy^2} \tag{12.23}$$

where, q is discharge per unit width of the channel.

Equation 12.22 can be plotted as shown in Figure 12.5 to show how the specific head H varies with the depth of flow y for progressively increasing values of discharges per unit width : q_1, q_2, q_3, etc. This diagram shows that two different depths can exist with a given specific head H and discharge q. In Figure 12.5 for example, the depth at A is small where the velocity is great, and the other depth at B is great where the velocity is small. These depths are termed alternate depths, because they can occur at the same specific head, but independently of each other, depending only upon the boundary conditions of the channel. Also of significance is the fact that there is a minimum value of specific head for a given discharge, such as at C_1, C_2 and C_3 in Figure 12.5.

It can be shown that this minimum specific head corresponds to the condition of a critical flow. Thus the depth of flow for the minimum value of the specific head H is equal to the *critical depth* y_c. In a rectangular channel, the critical depth can be evaluated by differentiating Equation 12.23 with respect to y and setting it equal to zero and rearranging to yield:

$$q = \sqrt{g y_c^3} \tag{12.24}$$

From Equations 12.22 and 12.24:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = 2\frac{V_c^2}{2g} = 2/3H \tag{12.25}$$

where V_c is the critical velocity.

For nonrectangular channels the equation for critical velocity V_c is:

$$V_c = \sqrt{\frac{gA_c}{K_e B_c}} \tag{12.26}$$

where A_c and B_c are, respectively, the cross section and top width of the critical flow, and K_e is the energy-flux correction coefficient.

(d) Discharge Diagram

When the discharge q in Equation 12.23 is plotted as a function of the depth of flow y for a constant specific head H, the resulting curve as shown in Figure 12.6 forms a discharge diagram. This curve indicates a maximum discharge q_{max}. By differentiating q in Equation 12.23, with respect to y and setting $dq/dy = 0$ it can be shown that this maximum discharge occurs at the critical-flow condition and is equal to:

$$q_{\max} = \sqrt{g(2/3H)^3} = \sqrt{g y_c^3} \tag{12.27}$$

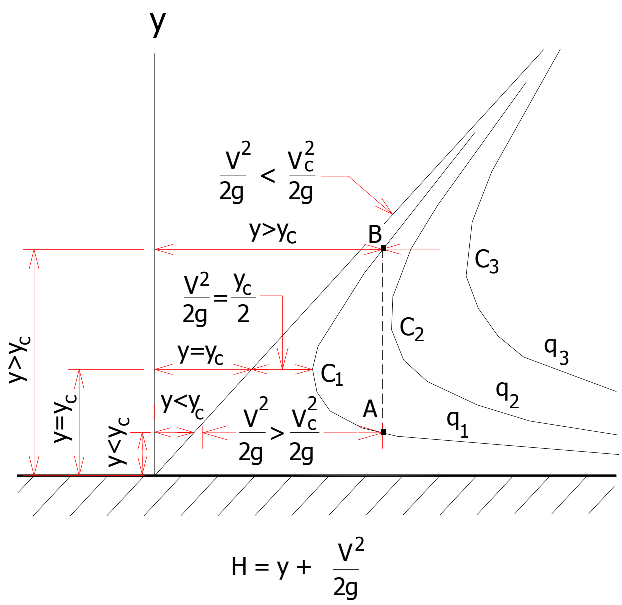


Figure 12.5 Specific-head Diagrams (Chow, 1964)

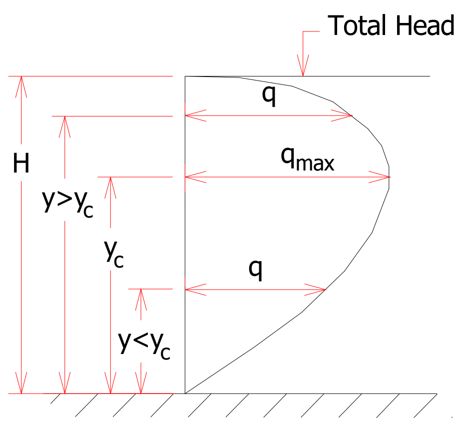


Figure 12.6 Discharge Diagram (Chow, 1964)

12.3.2 Nonuniform Flow Formula

There are two types of nonuniform flow. In one the changing conditions extend over a long distance, and this is called gradually varied flow. In the other the change may take place abruptly and the transition is thus confined to a short distance and this is designated as rapidly varied flow.

(a) Gradually Varied Flow

When the cross sections of flow in an open channel varies gradually along the channel so that the resulting changes in velocity take place very slowly, and thus the accelerative effects are negligible, the flow is known as gradually varied flow (Figure 12.4). The water surface of a gradually varied flow is called the flow profile, or commonly known as the backwater curve.

Changes in cross section of the flow may result either from a change in geometry of the channel, such as change in slope or cross-sectional shape, or an obstruction; or from an unbalance between the forces of resistance to retard the flow and the forces of gravity tending to accelerate the flow.

There are several types of flow profiles. In order to analyse these profiles, the total head H at a channel section can be expressed as:

$$H = K_e \frac{V^2}{2g} + y + z = K_e \frac{Q^2}{2gA^2} + y + z \qquad (12.28)$$

where K_e is the energy-flux correction coefficient, y is the depth of flow, z is the elevation of the channel bed above some arbitrary datum, Q is the discharge, and A is the cross section of the flow. Since the variation of these terms with distance x along the channel is desired, assuming $K_e = 1$, Equation 12.28 can be differentiated with respect to x to obtain:

$$\frac{dH}{dx} = -\frac{Q^2}{gA^3}\frac{dA}{dx} + \frac{dy}{dx} + \frac{dz}{dx} \qquad (12.29)$$

Let $dA = Bdy$, where B is the top width of the cross section of flow. Then:

$$\frac{dH}{dx} = -\frac{Q^2 B}{gA^3}\frac{dy}{dx} + \frac{dy}{dx} + \frac{dz}{dx} \qquad (12.30)$$

The gradient of total head dH/dx can be set equal to the negative of the slope obtained from the Chezy equation, or $S = (Q/A)^2/C^2R$, and the bed slope is equal to $dz/dx = -(Q/A_o)^2/C_o^2R_o = -S_o$ for uniform-flow conditions. The subscript o represents the uniform-flow condition. For simplicity, however, a wide rectangular channel can be assumed, so that $Q/B = q$ equal to the discharge per unit width and the hydraulic radius $R = A/B = y$. Equation 12.30 then becomes:

$$-\frac{q^2}{C^2y^3} = \frac{dy}{dx}\left(1 - \frac{q^2}{gy^3}\right) - \frac{q^2}{C_o^2Y_o^3} \qquad (12.31)$$

Furthermore, $q^2/g = y_c^3$, so that Equation 12.31 can be rearranged to solve explicitly for dy/dx, which is the rate of change of the depth of flow with respect to the distance along the channel. Thus:

$$\frac{dy}{dx} = \frac{q^2/C_o^2y_o^3 - q^2/C^2y^3}{1-(y_e/y)^3} \qquad (12.32)$$

which simplifies to:

$$\frac{dy}{dx} = S_o\frac{1-(C_o/C)^2(y_o/y)^3}{1-(y_C/y)^3} \qquad (12.33)$$

If the change in the Chezy C is not great from one point to another along the channel, the ratio C_o/C can be considered equal to 1.0. However, the Manning n is usually more nearly constant from section to section. Hence Equation 12.20 can be used in Equation 12.33 to yield:

$$\frac{dy}{dx} = S_o\frac{1-(n/n_o)^2(y_o/y)^{10/3}}{1-(y_c/y)^3} \qquad (12.34)$$

Using Equation 12.34 it is possible to classify the various flow profiles, which may occur in open channels.

(i) Classification of Flow Profiles

The analysis of flow profiles depends first upon the sign of dy/dx. If dy/dx is positive, the depth is increasing downstream, and if it is negative, the depth is decreasing downstream. From Equation 12.34 it can be seen that the slope dy/dx depends upon S_o, n/n_o, y_o/y and y_c/y. In the following analysis, it is assumed that $n/n_o = 1.0$. Although this assumption is not justified for all conditions, it may be taken as sufficiently accurate for the purpose of this analysis. Hence:

$$\frac{dy}{dx} = S_o\frac{1-(y_o/y)^{10/3}}{1-(y_c/y)^3} \qquad (12.35)$$

The slope of the channel serves as the primary means of classification. If the bed slope S_o is negative, the bed rises in the direction of flow. This slope is called an adverse slope, and the flow profiles over it are known as A profiles. If $S_o = 0$, the bed slope is horizontal and the profiles over it are H profiles. When $S_o > 0$, the bed slope may be mild, steep, or critical and the corresponding flow profiles are M profiles, S profiles, or C profiles, depending upon the ratio of y_o/y_c. When $y_o/y_c > 1.0$, an M profile exists; when $y_o/y_c = 1.0$, a C profile exists, and when $y_o/y_c < 1.0$, an S profile exists.

A further classification of flow profiles depends upon the ratios y_c/y and y_o/y. If both y_c/y and y_o/y are less than 1.0, then the profile is designated as type 1, for example,

M_1, S_1 and C_1. If the depth *y* is between the normal depth y_o and the critical depth y_c, then it is type 2, such as M_2, H_2, S_2, and A_2. If both y_c/y and y_o/y are greater than 1.0, then the profile is type 3, such as M_3, C_3, S_3, H_3 and A_3.

Various flow profiles are shown in Figure 12.7 where the longitudinal distance has been shortened and the slopes have been exaggerated for the sake of clarity. The general characteristics of the flow profiles are summarised in Table 12.1.

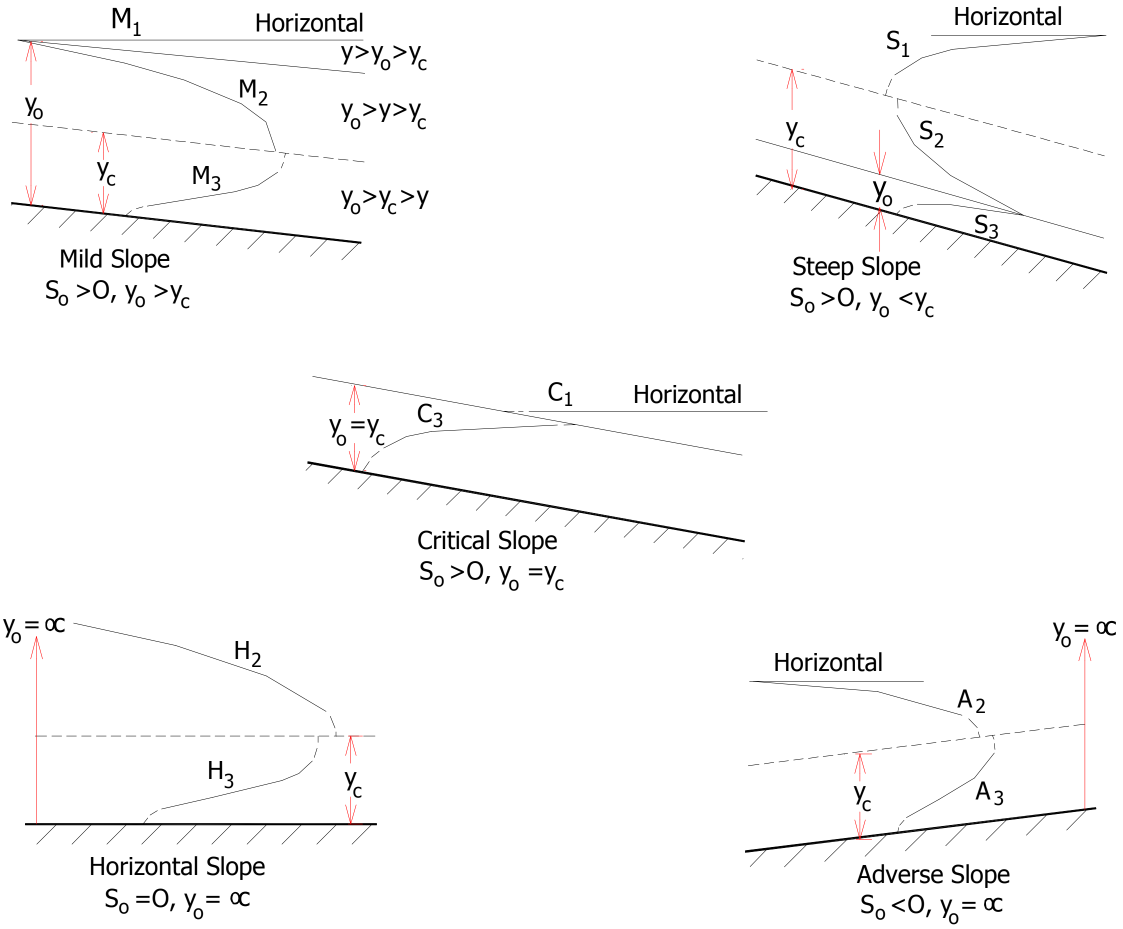


Figure 12.7 Classification of Flow Profiles.

Table 12.1 Characteristics of Flow Profiles.

Class	Bed Slope	$y{:}y_o{:}y_c$	Type	Symbol
Mild	$S_o > 0$ $S_o > 0$ $S_o > 0$	$y > y_o > y_c$ $y_o > y > y_c$ $y_o > y_c > y$	1 2 3	M_1 M_2 M_3
Critical	$S_o > 0$ $S_o > 0$	$y > y_o = y_c$ $y < y_o = y_c$	1 3	C_1 C_3
Steep	$S_o > 0$ $S_o > 0$ $S_o > 0$	$y > y_c > y_o$ $y_c > y > y_o$ $y_c > y_o > y$	1 2 3	S_1 S_2 S_3
Horizontal	$S_o = 0$ $S_o = 0$	$y > y_c$ $y_c > y$	2 3	H_2 H_3
Adverse	$S_o < 0$ $S_o < 0$	$y > y_c$ $y_c > y$	2 3	A_2 A_3

(ii) Computation of Backwater Curves

By integrating Equation 12.35 a mathematical relation can be obtained to represent the surface profile of a gradually varied flow. For practical purposes, however, a step method, described below is widely used.

Figure 12.8 illustrates a channel of length ΔL which is sufficiently short so that the water surface can be approximated by a straight line. By geometry or from Equation 12.29 it can be shown that $\Delta H/\Delta L = S_o - S$ or:

$$\Delta L = \frac{\Delta H}{S_o - S} \tag{12.36}$$

where H is the specific head; $S = -dH/dx$, the average energy gradient; and $S_0 = -dz/dx$. S_o is the slope of the channel bed.

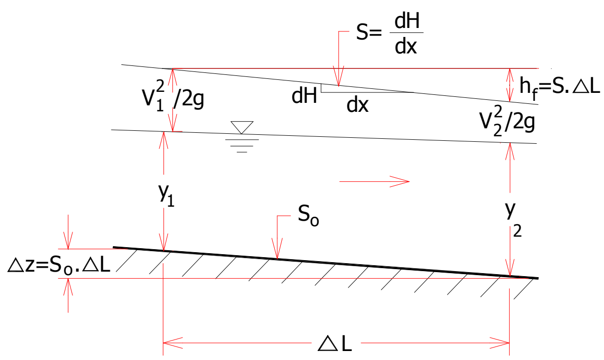


Figure 12.8 Definition Sketch for Computation of Backwater Curves

The energy gradient at a channel section can be computed by the Manning equation as $S = Q^2n^2/(AR^{2/3})^2$. The average of the energy gradients at the two end sections of the reach is used for S in Equation 12.36.

The step method is characterised by dividing the channel into short reaches and applying Equation 12.36 by steps from one end of a reach to the other. To start the computation, the depth of flow at the beginning section should be given or assumed. From a given discharge and channel conditions, the specific heads at the two end sections and their difference ΔH and the energy slope at the two end sections and their average are computed. Substituting these quantities and the channel slope S_o in Equation 12.36, the length of the reach is computed. By repeating the computation for the subsequent reaches, the entire flow profile or backwater curve can be determined. It should be noted that the step computation should be carried upstream if the flow is tranquil and downstream if the flow is rapid. If carried in the wrong direction, the computation tends inevitably to make the result diverge from the correct flow profile. For a comprehensive treatment of the computation of flow profiles see Chow (1959).

(b) Rapidly Varied Flow

Rapidly varied flow, on the other hand, produces abrupt changes in depth and velocity over very short distances, as in the case of flow over an emergency spillway, through a hydraulic jump, or beneath a sluice gate. Rapidly varied flow usually involves wave phenomena, which preclude the use of uniform flow formulas. Nonuniform flow can also be unsteady, as in the passage of a runoff peak or flood wave through a stormwater drain or man-made channel.

The hydraulic jump is a rapidly varied flow phenomenon in which flow in a channel changes abruptly from rapid/supercritical flow at a relatively shallow depth (less than y_c) to tranquil/subcritical flow at a greater depth (greater than y_c). The depth before the jump is called initial depth, while the depth after the jump is known as the sequent depth. The situation is illustrated in Figure 12.9.

The hydraulic jump may be employed as a device for the dissipation of excess energy, as where a steep drain enters a larger drain at a junction. In stormwater projects, the hydraulic jump may be used to consume excess energy and avoid scour of earthen channels. Thus, the analysis of hydraulic jumps usually has three objectives. First, the location of the jump is important because of the potential of unexpected surcharges or channel scour. This can be determined by searching for pipe/channel elements where the flow is supercritical upstream and subcritical downstream. Once this is determined, it is important to compute the two depths, y_1 and y_2, which are the initial and sequent depths, respectively. Third, the energy loss H_L dissipated by the jump is often an important design consideration. The pertinent depth equation for a rectangular channel section is:

$$\frac{y_2}{y_1} = 0.5\left[\left(1 + 8F_1^2\right)^{1/2} - 1\right] \tag{12.37}$$

in which F_1 is the Froude Number at the upstream section. The energy lost in the jump, H_L is obtained by subtracting the specific energy at section 2 in Figure 12.9 from that at section 1:

$$H_L = H_1 - H_2 = \frac{(y_2 - y_1)^3}{4y_1y_2} \tag{12.38}$$

12.4 STEADY PRESSURE PIPE FLOW

Steady flow in closed conduits involves a combination of uniform or nonuniform flow, laminar or turbulent flow and flow over smooth or rough boundaries.

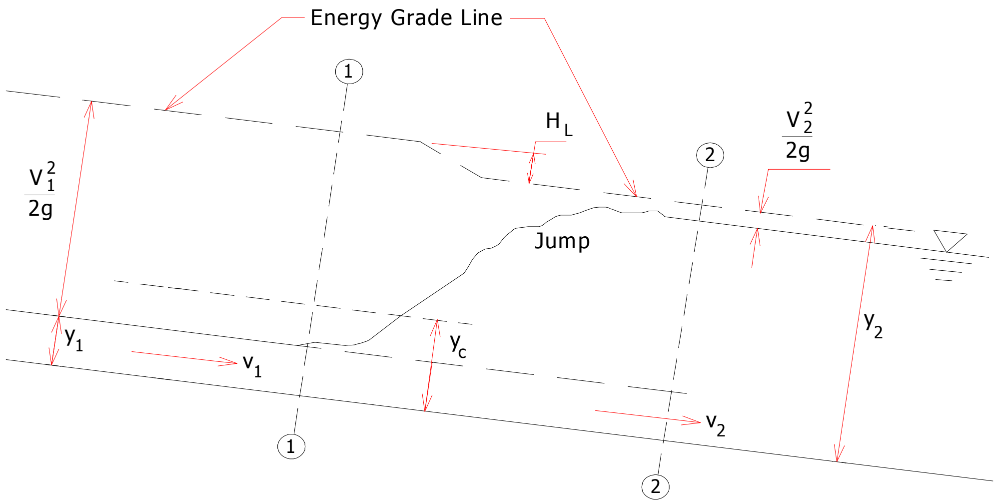


Figure 12.9 Hydraulic Jump.

At the upstream end of a pipe there is a region of flow development in which the boundary layer is developing and the flow is technically nonuniform. Therefore the velocity distribution changes from section to section.

12.4.1 Uniform Flow

Problems involving steady uniform flow in closed conduits may be solved by the energy equation, (Equation 12.14) which is written for two sections, 1 and 2 as follows, assuming K_e = 1.00 :

$$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2+H_L \tag{12.39}$$

where H_L is the sum of the losses caused by both the shear resistance h_f and the pressure resistance h_L ; that is, $H_L = h_f + h_L$. The shear resistance can be evaluated by the Darcy-Weisbach equation:

$$h_f=f\frac{L}{D}\frac{V^2}{2g} \tag{12.40}$$

where *f* is a resistance coefficient, *L* is the length of the pipe, *D* is the diameter, *V* is the mean velocity of flow and *g* is the gravitational acceleration. The resistance coefficient depends upon the Reynolds number of flow and the relative roughness *e/D*, where e is the average size of the roughness element. For laminar flow or for turbulent flow with a smooth surface, the relative roughness is unimportant and hence *f* depends on Re alone. For a rough boundary, Re is unimportant and then f depends on *e/D* alone. The relationship between *f*, Re and *e/D* is related by the so-called Moody resistance diagram. In this diagram the roughness e for various pipe materials and inside coatings is given (Chow, 1964). The average value of the range of *e* should be used unless additional information gives reason to use the smaller or larger values of the range. However, it may be seen from the diagram that a rather larger error in the estimate of e would result in a smaller error in f.

When Re is less than 2,000 the flow is laminar and f = 64/Re. When Re increases, the laminar sublayer is penetrated by roughness elements and the flow becomes turbulent. The region between Re = 2,000 to approximately Re = 3,500 indicates an indefinite transition for flow to change from laminar to turbulent. For turbulent flow, the resistance coefficient can be estimated from the following equations:

For turbulent boundary layer over smooth boundary:

$$\frac{1}{\sqrt{f}}=2\log\left(\mathrm{Re}\sqrt{f}\right)-0.8 \tag{12.41}$$

For turbulent boundary layer over rough boundary:

$$\frac{1}{\sqrt{f}}=2\log\frac{D}{e}+1.14 \tag{12.42}$$

For the transition from smooth to rough boundary, the above two equations can be combined to produce the following semiempirical form:

$$\frac{1}{\sqrt{f}}=1.14-2\log\left(\frac{e}{D}+\frac{9.35}{\mathrm{Re}\sqrt{f}}\right) \tag{12.43}$$

which is known as the Colebrook-White equation. This equation reduces to Equation 12.42 for flow in smooth pipes and to Equation 12.43 for flow in rough pipes. Pipes having a noncircular cross section but a simple geometrical shape, such as a rectangle, a trapezoid, or an ellipse which does not differ markedly from circular, can be solved by Moody diagram if the hydraulic radius equivalent to that of a circular pipe is used. Thus R = D/4, or D = 4R and Equation 12.40 becomes h_f = f (L/4R) (V^2/2g). For turbulent flow, this use of hydraulic radius gives reasonably accurate results. For laminar flow, however, it gives increasingly inaccurate results as the shape of conduit differs more and more from circular.

12.4.2 Nonuniform Flow

In nonuniform flow the changes in velocity result in a change in momentum flux, which is accomplished only by pressures against the fluid in addition to the pressures, which would be associated with uniform flow. When such changes in velocity occur, zones of separation and secondary flow frequently result, and this consequently increases the shear and the turbulence at the expense of the piezometric head. Hence head losses h_L result. Since the foregoing changes in velocity and the resulting head losses are caused by nonuniform distribution of pressures on the boundary, the losses are termed structural/form losses because of pressure resistance and the associated changes (usually increases) in shear resistance. The form losses can be expressed as:

$$h_L = K\frac{V^2}{2g} \qquad (12.44)$$

where K is called the form-loss coefficient, and V is mean velocity of flow. Chapter 25 provides various kinds of form losses. These form losses are sometimes called minor losses. Such a term represents the true situations literally when the pipeline is relatively long and the friction loss coefficient f (L/D) in Equation 12.46 is large by comparison with K. For shorter pipe however, the form losses caused by pressure resistance may be of major importance.

(a) Compound Pipe

The principles presented in all the forgoing discussion can be used in combination to solve problems involving compound pipe. Figure 12.10 is an example of a compound pipe which consists of an entrance, a sudden expansion, a sudden contraction, a manhole, a bend, a gradual expansion, an outlet, and pipes of different diameters. Each of these items involves a head loss. The straight pipe involves friction resistance, and each of the others involves both shear and pressure resistance to make up the form losses. The energy equation may be written for any reach of pipe between sections a and b:

$$\frac{V_a^2}{2g}+\frac{p_a}{\gamma}+z_a=\frac{V_b^2}{2g}+\frac{p_b}{2g}+z_b+h_L \qquad (12.45)$$

If the upstream inlet is chosen as section a and the downstream reservoir as b, then h_L is the sum of all the losses indicated in Figure 12.10, or:

$$\begin{aligned} h_L(\textit{total loss}) = {} & h_{L01}(\textit{entrance loss}) + h_{f1}(\textit{pipe loss}) + \\ & h_{L12}(\textit{expansion loss}) + h_{f2}(\textit{pipe loss}) + \\ & h_{L23}(\textit{contraction loss}) + h_{f3}(\textit{pipe loss}) + \\ & h_{L3}(\textit{manhole loss}) + h_{f4}(\textit{pipe loss}) + \\ & h_{L45}(\textit{bend loss}) + h_{f5}(\textit{pipe loss}) + \\ & h_{L57}(\textit{gradual expansion loss}) + h_{L67}(\textit{exit loss}) \end{aligned} \qquad (12.46)$$

Each of the losses must be determined by the methods already discussed, and then added together to get h_L.

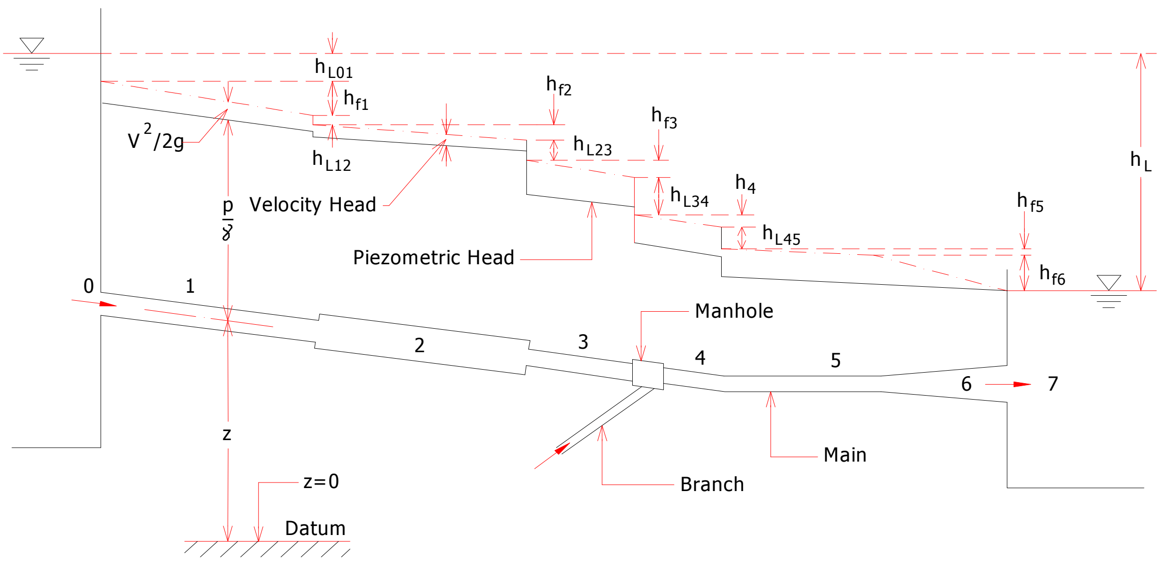


Figure 12.10 Energy Diagram for Compound Pipe.

(b) *Branching Pipes*

Figure 12.11 illustrates a branching pipe system and indicates that the flow into the junction must equal the flow out of the junction. Furthermore the piezometric head at the junction is common for all three pipes. The three piezometric readings at A, B and C can be considered as the water-surface elevations in three inlets/reservoirs, as shown by broken lines, since the velocity head is considered as insignificant in these problems when compared with the head losses due to boundary resistance.

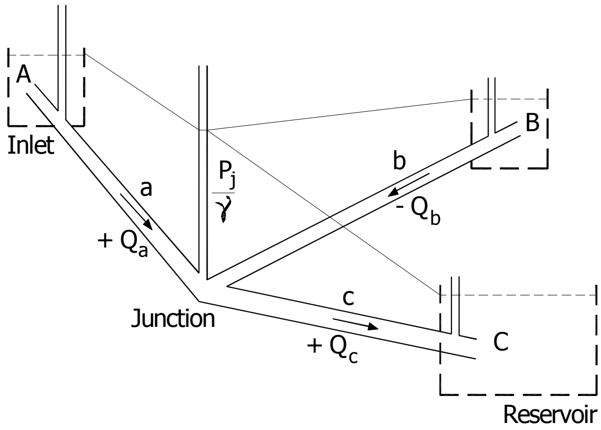


Figure 12.11 Branching Pipes

There are three different flow conditions for the continuity equation, any one of which may be applicable for a given problem. Each flow condition depends upon the slope of the hydraulic gradient as follows:

1. flow from pipe a into pipes b and c, so that the piezometric head line for pipe b slopes downward to the right and $Q_a = Q_b + Q_c$
2. flow from pipes a and b into pipe c, so that the piezometric head line for pipe b slopes downward to the left and $Q_a + Q_b = Q_c$
3. flow from pipe a into pipe c, with no flow in pipe b, so that the piezometric head line for pipe b is horizontal and $Q_a = Q_c$ while $Q_b = 0$.

12.5 UNSTEADY SHALLOW SURFACE FLOW

All hydraulic routing principles (in Chapter 14) and some important computer models (Chapter 17) involve solution of unsteady flow equations. The types of free surface flows discussed in this section are the overland, floodplain/open channel/partial pipe and pond/reservoir. Their governing unsteady flow equations are derived based on general development of the continuity and momentum equations with continuous vertical and lateral inflows.

For the control volume shown Figure 12.12 the following equations of motion in the x-direction are obtained:

Continuity equation:

$$\frac{\partial y}{\partial t}+V\frac{\partial y}{\partial x}+y\frac{\partial V}{\partial x}=(i-f)+\frac{2q_L}{b} \qquad (12.47)$$

Momentum equation:

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial y}{\partial x}=\left\{(i-f)+\frac{2q_L}{b}\right\}\frac{V}{y}-\left(1+\frac{2y}{b}\right)\frac{\tau_0}{\rho y}+g\theta \qquad (12.48)$$

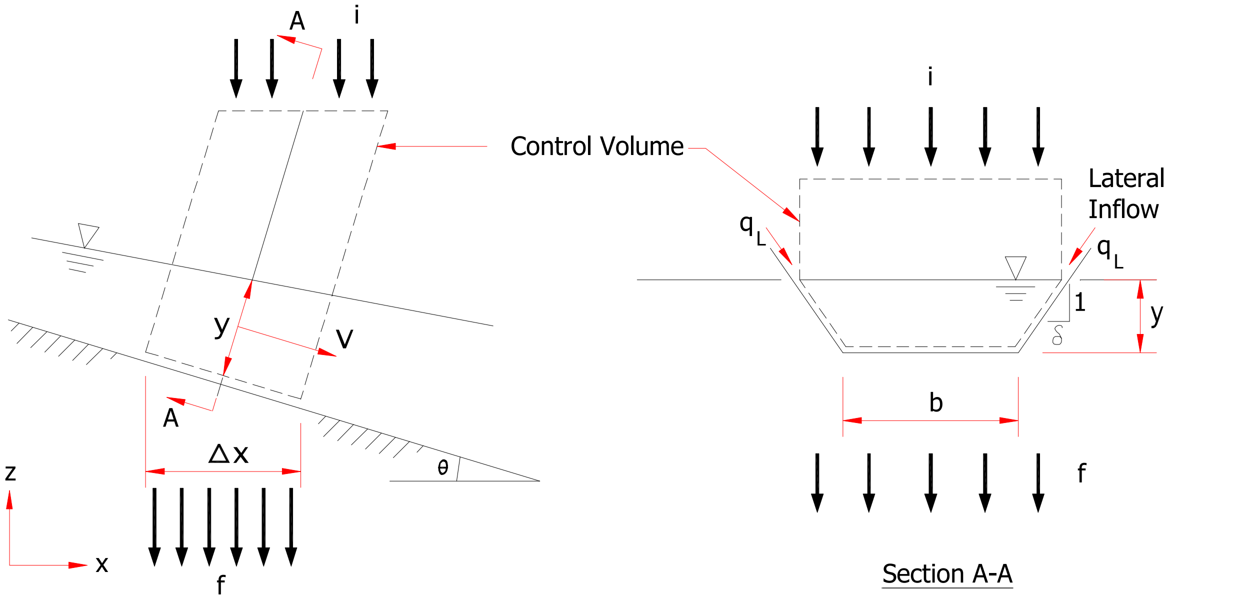


Figure 12.12 Control Volume for Shallow Surface Flow (Eagleson, 1970)

where

- i = rainfall intensity
- f = infiltration rate
- q_L= lateral inflow
- θ = slope
- τ_0= bed shear
- b = bottom width
- ρ = density
- g = gravity acceleration

These equations are based on shallow water, small bottom slope and uniform velocity distribution.

12.5.1 Kinematic Waves Equation

For overland flow (Figure 12.13), kinematic wave equation is valid and applied in which the inflow, free surface slope and inertia terms of the momentum equation are all negligible in comparison with those of bottom slope and friction. For a typical land surface (from grass to tar or gravel), there will be fluctuations in depth and roughness such that the flow regime may vary from laminar to turbulent. The solution of these overland flow problems is thus contained in the following continuity and momentum equations, respectively (Lighthill and Whitham, 1955):

$$\frac{\partial y}{\partial t}+\frac{\partial q}{\partial x}=(i-f)+\frac{2q_L}{b} \qquad (12.49)$$

$$q=\alpha y^m \qquad (12.50)$$

Where α and m can be derived under laminar or turbulent flow conditions.

The dynamic uniformity of this approach precludes solution which exhibit changes in surface profile due to dynamic variations, thus Froude Number can be greater than or less than unity. Boundary and initial conditions apply only to solution of the continuity equation; therefore, changes in water surface profile will be caused only by changes in local flow rate and will be transmitted in the direction in which a kinematic wave propagates.

12.5.2 Dynamic Waves Equation

A one-dimensional dynamic equation applies to channel or partial pipe flow. For flow through open channel, natural/infiltrating, the rainfall runoff process has negligible effect upon the flow dynamics, hence (*i-f*) term can be omitted from the momentum equation. Lateral inflow, however, is important if the width of the catchment is large with respect to depth of the channel.

In channel flow both the inertia and pressure forces are important and if inflow terms are negligible and for a wide channel/shallow water the following equations govern:

Continuity equation:

$$\frac{\partial y}{\partial t}+V\frac{\partial y}{\partial x}+y\frac{\partial V}{\partial x}=0 \qquad (12.51)$$

Momentum equation:

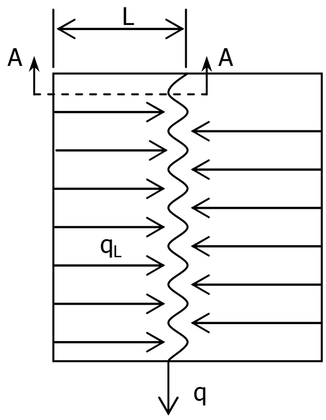
$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial y}{\partial x}=-\frac{\tau_0}{\rho y}+g\theta \qquad (12.52)$$

For flow in vegetated drain/small stream, the class of channelised flows to which the only significant inputs, continuous along the stream axis, are rainfall, infiltration and overland flow. The continuity equation for small stream can be written as:

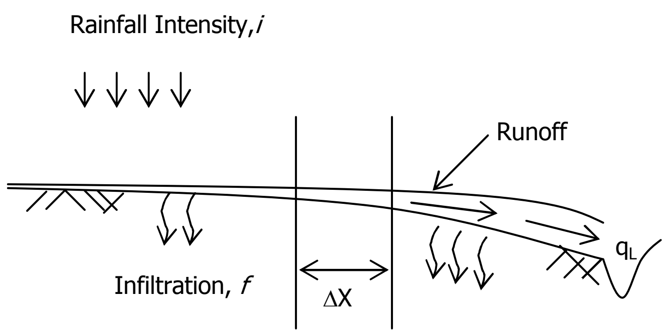
$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=(i-f)(b+2\delta y)+2q_L \qquad (12.53)$$

where,

- A = flow cross-section
- Q = discharge
- δy = slope (Figure 12.12)



(a) Plan



(b) Section A-A

Figure 12.13 Overland Sheet Flow

12.5.3 Hydrodynamics of Ponds and Small Reservoirs

Small water bodies where ratio of depth over horizontal dimension is much less than 1.0 are considered shallow and they are subject to circulation created by inflow-outflow and wind inputs in urban stormwater processes. Water flows in a shallow pond usually predominant in horizontal plane and variation of velocity and density in vertical direction are small enough to be neglected. It is thus adequate to adopt the depth averaged (vertically integrated) two-dimensional dynamic equations to solve this field problems (see Chapter 14).

12.6 POROUS MEDIA FLOW

12.6.1 General

The techniques of stormwater infiltration/retention analysis are based on understanding of the physical processes mathematically. The basic law of flow is based on Darcy's law. When it is put together with an equation of continuity that describes the conservation of fluid mass inflow through a porous medium a differential equation results. In this section the equations of steady or transient flows for both unsaturated and saturated media are presented.

12.6.2 Darcy's Law

Darcy's law was founded based on saturated sand column experiment (Darcy, 1856) as illustrated in Figure 12.14. Under steady condition/macroscopic section specific discharge or Darcy's velocity/Darcy's flux:

$$V = -K\frac{\Delta h}{\Delta l} \quad (12.54)$$

or

$$V = -K\frac{dh}{dl} \quad (12.55)$$

Total discharge (flow) through the column is:

$$Q = -AK\frac{dh}{dl} \quad (12.56)$$

where,

A = cross-sectional area of flow
V = Darcy's velocity
K = hydraulic conductivity
$\Delta h/\Delta l$ = hydraulic gradient
Δl = distance

Microscopic or pore velocity:

$$v_n = \frac{V}{n} \quad (12.57)$$

where n = porosity.

This indicates that for a sand with a porosity of 33 %, $v_n = 3V$. To define the actual flow velocity, one must consider the microstructure of the material.

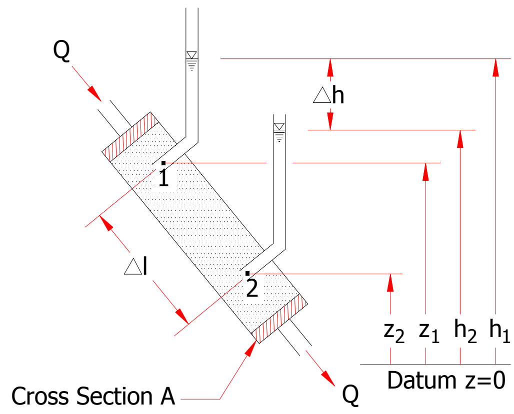


Figure 12.14 Experimental Illustration of Darcy's Law.

Darcy's law is valid for porous media flow in any direction, saturated or unsaturated and steady or transient

The hydraulic head or fluid potential $h = z + \psi$ is basic to an understanding of porous media flow and is a classical formulation of energy conservation or Bernoulli equation. Total head:

$$H = z + \psi + \frac{V^2}{2g} \quad (12.58)$$

This equation is applicable for steady saturated or unsaturated flows, just as for steady flow in conveyances

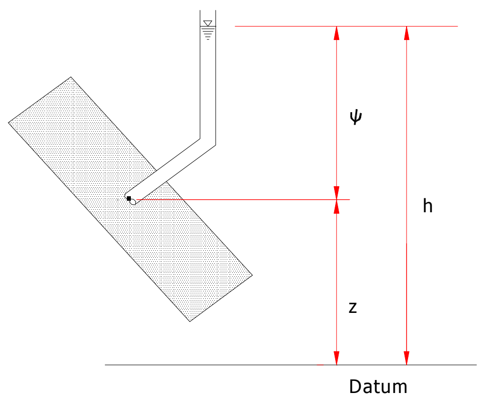


Figure 12.15 Darcy Manometer

12.6.3 Unsaturated Flow

(a) Steady Recharge Rate

In vadose zone it reflects that the water is held in media pores under surface-tension forces with pressure head $\psi < 0$ and termed as suction head or negative (-ve) pressure head. Measurement of suction head is obtained using tensiometers (Figure 12.16c). In unsaturated flow both moisture content θ and hydraulic conductivity K are functions of ψ. $K = K(\psi)$, $\theta = \theta(\psi)$, and $K = K(\theta)$

Darcy's flux for steady vertical unsaturated flow in isotropic media is:

$$V = -K(\psi)\frac{\partial(\psi + z)}{\partial z} \tag{12.59}$$

(b) Transient Flow

For general three-dimensional flow in an elemental control volume the equation of continuity gives (Richards, 1931):

$$\frac{\partial}{\partial x}\left[K(\psi)\frac{\partial \psi}{\partial x}\right]+\frac{\partial}{\partial y}\left[K(\psi)\frac{\partial \psi}{\partial y}\right]+\frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z}+1\right)\right] = C(\psi)\frac{\partial \psi}{\partial t} \tag{12.60}$$

where $C(\psi)$ is the specific moisture capacity $\partial\theta/\partial\psi$

In one-dimensional form (z-direction) the Equation 12.60 reduces to:

$$\frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z}+1\right)\right] = C(\psi)\frac{\partial \psi}{\partial t} \tag{12.61}$$

or in independent variable θ it is:

$$\frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z}+K\right) = \frac{\partial \theta}{\partial t} \tag{12.62}$$

where D is the soil water diffusivity, $K(\partial\psi/\partial\theta)$ or $K(\psi)/C(\psi)$.

12.6.4 Saturated Flow

(a) Steady Recharge Rate

The following equation is derived for steady shallow infiltration and for filtration that have achieved saturated conditions in a homogeneous porous column (Figure 12.17):

$$q_{1-2} = AK\frac{\left(\frac{P_1}{\gamma}+Z_1\right)-\left(\frac{P_2}{\gamma}+Z_2\right)}{Z_1 - Z_2} \tag{12.63}$$

$$\text{or} \quad = AK\frac{\Delta h}{\Delta Z} \tag{12.64}$$

where,

q = flow
K = saturated hydraulic conductivity
A = cross-sectional area of flow
$\Delta h/\Delta Z$ = hydraulic gradient

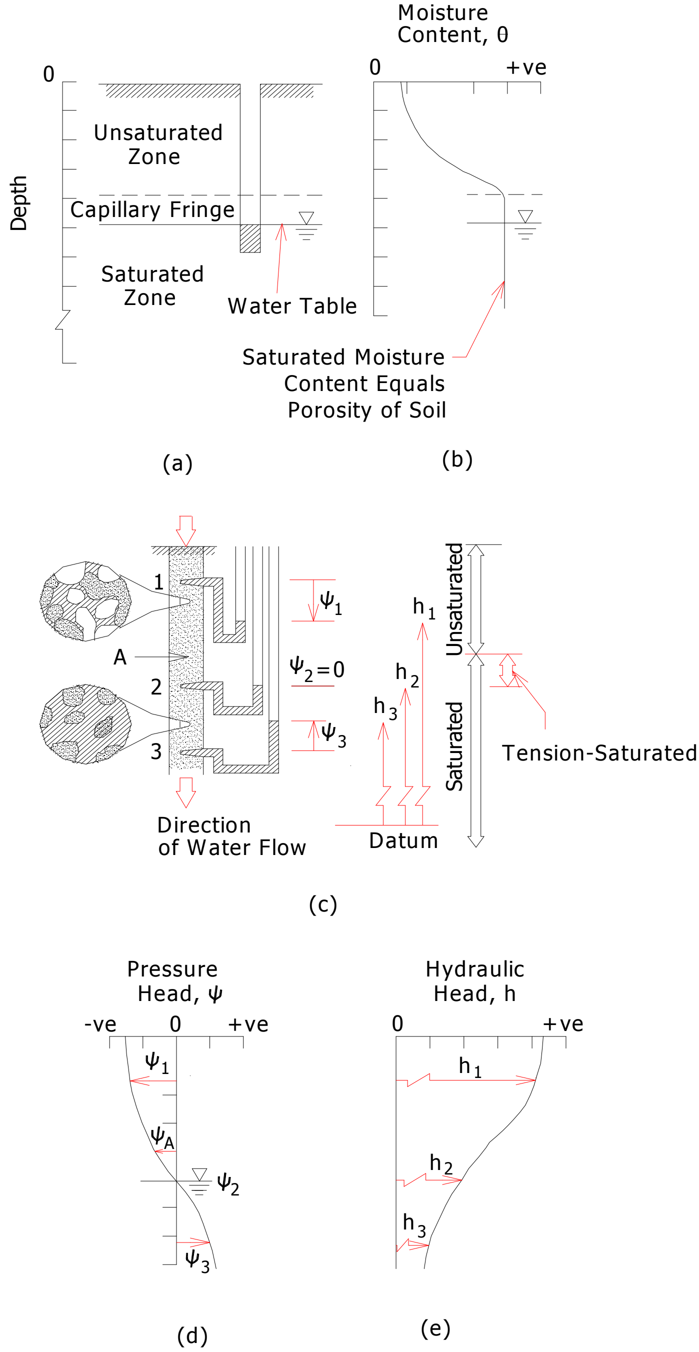


Figure 12.16 Unsaturated Zone Conditions (Freeze and Cherry, 1979)

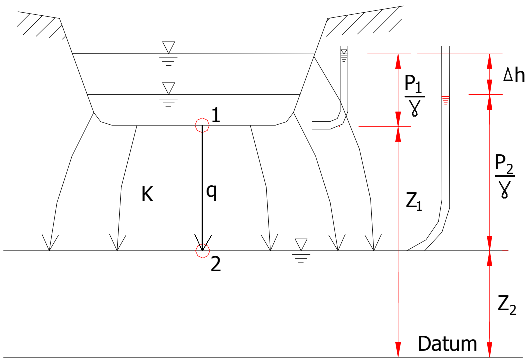


Figure 12.17 Saturated Flow

(b) Infiltration/Recharge into Phreatic Aquifers

Stormwater infiltration or recharge, in steady or transient conditions, into underlying phreatic or unconfined aquifers can have impact on the water table (Figure 12.18).

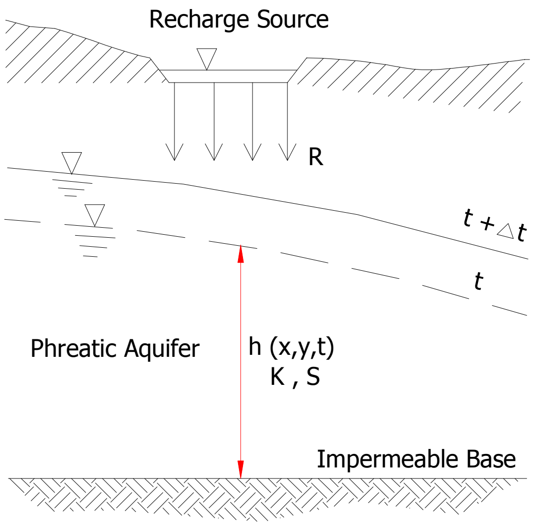


Figure 12.18 Recharge into Phreatic Aquifer

The water table or the aquifer storage capability to react with the designed infiltration/recharge magnitude can be analysed using the following nonlinear partial differential equation (PDE) in general two-dimensional form (Boussinesq, 1904):

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kh\frac{\partial h}{\partial y}\right)+R=S\frac{\partial h}{\partial t} \quad (12.65)$$

The Equation 12.65 is nonlinear because of $h\partial h/\partial x$ and for possible solution it is linearised into h^2, forming the following PDE:

$$\frac{\partial^2 h^2}{\partial x^2}+\frac{\partial^2 h^2}{\partial y^2}+\frac{2R}{K}=\frac{S}{T}\frac{\partial h^2}{\partial t} \quad (12.66)$$

where

- T = $K\,\bar{h}$ (Transmissivity)
- R = infiltration /recharge
- S = storativity or specific yield S_y
- K = saturated conductivity

The equation is valid for impermeable or clay horizontal base. In practical situation sometime however the base is semipermeable or leaky and in such case leakage flow, upward or downward, be incorporated into the equation. Solution of Equation 12.66, analytical or numerical, is usually based on Dupuit Forcheimer assumptions.

12.6.5 Steady Well Flow Hydraulics

Hydraulic of radial flow is important in the analysis and design of stormwater recharge using wells. In most literatures these fundamental hydraulics are not readily available. However, hydraulics of pumping wells are numerous, applied to both phreatic and confined aquifers under steady or transient conditions (Bear, 1979). To some extent the pumping well hydraulics can be applied to recharge well, through inverted approximation.

For steady flow of recharge into confined aquifer (Figure 12.19) the equation is:

$$Q_r = -2\pi\, r\, K(r)\, B\,\frac{d\phi}{dr} \quad (12.67)$$

where Q_r is the constant rate of recharge. The build up/non-build up steady state expressions are respectively, recommended as:

$$\phi(r_w)-\phi(R)=\frac{Q_r}{2\pi B}\left\{\int_{r_w}^{r_e}\frac{dr}{rK(r)}+\ln\frac{R}{r_e}\right\} \quad (12.68)$$

$$\phi(r_w)-\phi(R)=\frac{Q_r}{2\pi BK_o}\ln\frac{R}{r_w} \quad (12.69)$$

which corresponds to $K=K(r)$ for $r_w < r < r_e$ and a constant $K=K_0$ for the entire region $r_w < r < R$. R is the radius of influence where practically no build-up is observed. The additional build-up thus obtained is due to clogging. When the permissible build-up is limited, this means that the recharge rate Q_r has to be reduced. When the reduced recharge rates become uneconomic, cleaning operations have to be undertaken in order to restore the recharge capacity of the well.

A natural, approximately uniform, flow exists in most undisturbed aquifers. When recharging wells are introduced with such flows, the method of superposition (Bear, 1979) may be employed to determine the resulting flownets (Figure 12.20).

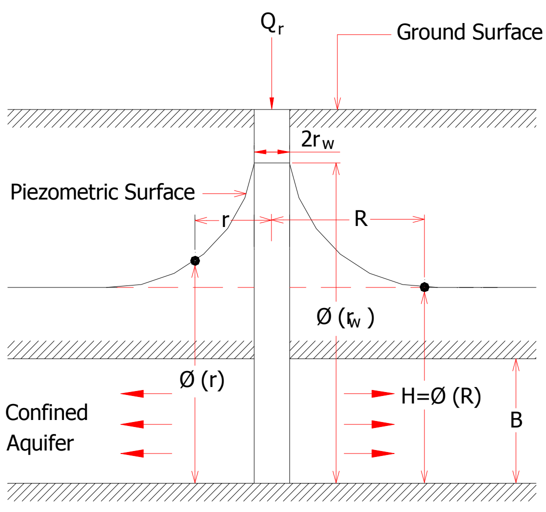


Figure 12.19 Steady Recharge into Confined Aquifer

A uniform flow at a constant discharge q_o in the x-direction takes place in the aquifer of thickness B and transmissivity T. The piezometric head/potential distribution ϕ and the stream function ψ as a result of single application of recharge Q_r located at x = 0 and y = 0 are given by:

$$\phi = -\frac{q_o}{K}x - \frac{Q_r}{4\pi T}\ln\left(x^2 + y^2\right) \tag{12.70}$$

$$\psi = -\frac{q_o}{k}y - \frac{Q}{2\pi T}\tan^{-1}\left(\frac{y}{x}\right) \tag{12.71}$$

The velocity components V_x , V_y in the +x and +y directions, respectively are given by:

$$V_x = \frac{q_o}{n} + \frac{Q_r x}{2\pi n B\left(x^2 + y^2\right)} \tag{12.72}$$

$$V_y = \frac{Q_r y}{2\pi n B\left(x^2 + y^2\right)} \tag{12.73}$$

For a recharge well of finite radius r_w:

$$\phi = -\frac{q_o B}{T}x - \frac{Q_r}{2\pi T}\ln\frac{r}{r_w} \tag{12.74}$$

For multiple and partially penetrated recharge well the same hydraulic principles used in pumping wells is equally applied (Bear, 1979).

12.6.6 Integrated Flow System

An integrated flow, coupling unsaturated-saturated equations, is useful in the planning and design of comprehensive stormwater recharge schemes over a regional urban groundwater basin. The transient influences are felt most strongly near the surface of the vadose zone. Freeze (1971) described the integrated mathematical model using unsaturated and saturated flow equations.

The model is also important for use in the planning of subsurface drainage facilities especially in vulnerable hillslope zones.

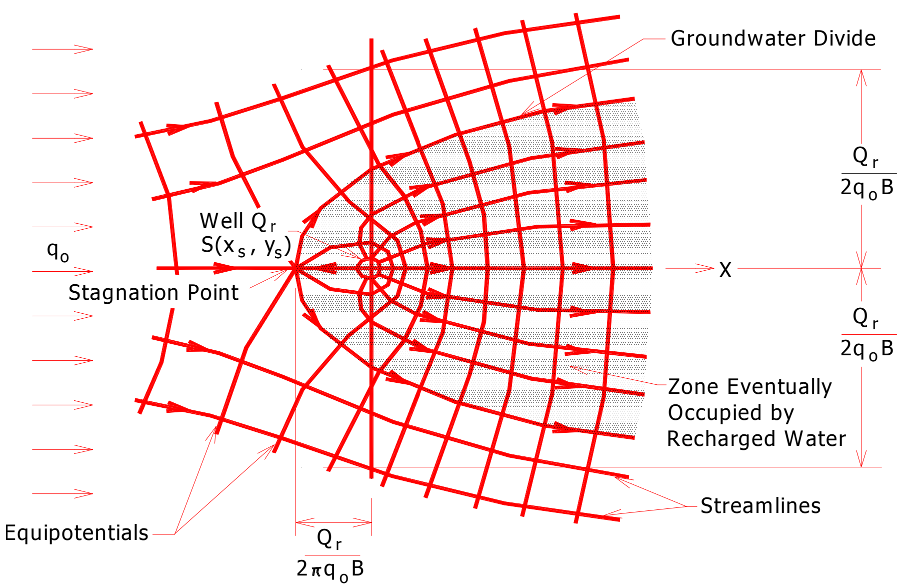


Figure 12.20 A Single Recharging Well in Uniform Flow

12.7 POLLUTANT TRANSPORT AND SETTLING PROCESSES

In order to investigate environmental concerns, stormwater specialists are usually called upon to study the transport, settlement and retention of various substances in surface water and porous media. These substances, referred to as constituents, may be contaminants, pollutants, artificial tracers, solid particles or other materials. The motion, spreading and settling of a mass may be due to advection, diffusion and dispersion. In this section, we shall first define commonly used terms and then present the equation of transport of a constituent as well as particulate settling in a fluid.

12.7.1 Transport Processes

(a) Definitions

The amount of substance in water is specified by the concentration C, which is defined as the mass of substance per unit volume of water. A constituent is said to be conservative if it does not decay, is not absorbed or adsorbed, and does not undergo chemical, biological, or nuclear transformation.

The transport of a constituent due to bulk motion of the fluid is called advection. Dispersion caused entirely by the motion of the fluid is referred to as mechanical dispersion, and that mainly due to concentration gradient is called diffusion. A combination of diffusion and mechanical dispersion is called hydrodynamic dispersion. The spreading of the constituent and its resulting dilution is due to hydrodynamic dispersion. In order to illustrate these concepts, let us consider uniform, laminar flow through a pipe. The velocity distribution in this flow at a cross section is parabolic. Let a substance be introduced across the pipe cross section. Due to higher flow velocity at the centre of the pipe, the substance will be carried to a greater distance near the centre than near the walls. Thus, the material will be dispersed due to nonuniform velocity distribution (Figure 12.21(a)).

To illustrate different processes, let us consider steady uniform flow through a pipe. Let the flow velocity be U and let the concentration of a constituent be initially zero. Let us assume that at time t_o, we introduce at the upstream end of the pipe a constituent such that concentration C_o is maintained at the pipe entrance. Let us designate the concentration at any location in the pipe by C. In order to plot the results in nondimensional form, we will use relative concentration, C/C_o. The time variation of C/C_o will plot as a step function, as shown in Figure 12.21(b). If the constituent is conservative and there is no dispersion and diffusion, then the constituent will propagate as plug flow, as shown by the vertical dotted line in Figure 12.21(c). However, due to dispersion and diffusion, the relative concentration at the outlet of the constituent front will first appear at time t_1. If we plot the relative concentration at different times as the front moves through the pipe, it will appear as shown in Figure 12.21(d). Due to mechanical dispersion and molecular diffusion, some of the constituent particles move faster than the average flow velocity, while others move slower.

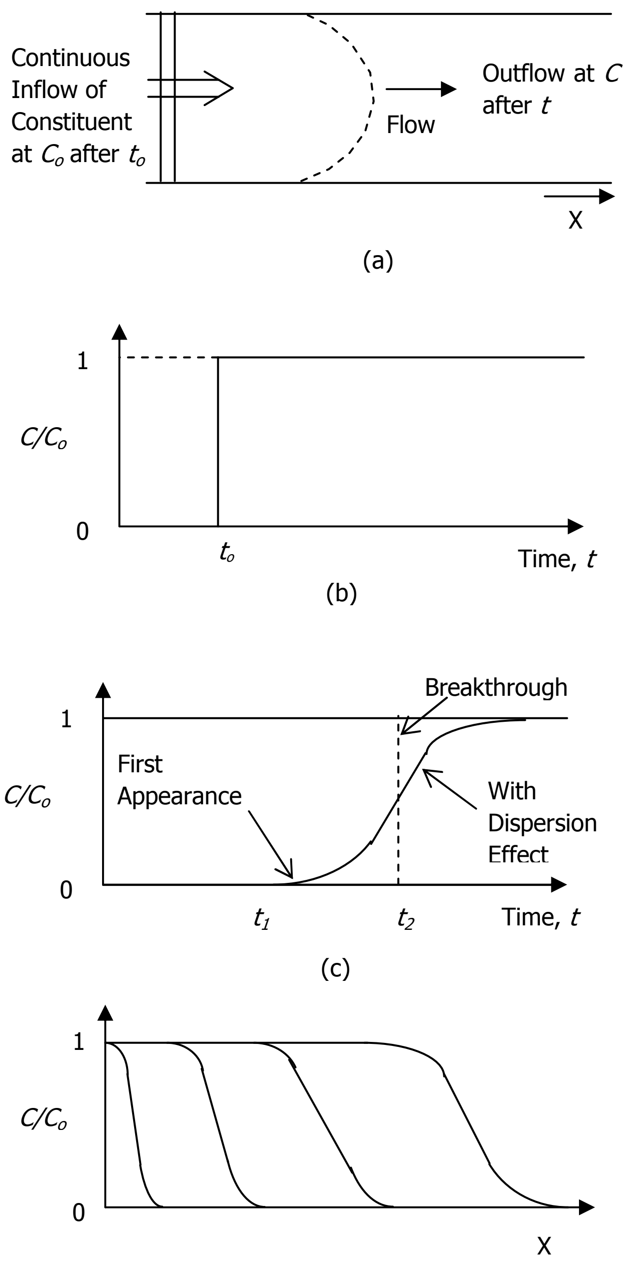


Figure 12.21 Dispersion in One-dimensional Flow (After Freeze and Cherry, 1979)

The mass of diffusing constituent per unit time passing through a given cross section in a stationary fluid is proportional to the concentration gradient. This is known as Fick's First Law and may be expressed as:

$$F = -D\frac{dC}{dx} \quad (12.75)$$

where,

F = mass flux per unit time per unit area

D = diffusion coefficient

C = constituent concentration

dC/dx = concentration gradient

Fick's law is based on molecular transport and states that a substance tends to equalise its distribution; i.e., it flows from a zone of high concentration to a zone of low concentration.

The advection equation for a conservative substance may be written as:

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = O \quad (12.76)$$

in which U = mean fluid velocity.

The Peclet number is the ratio of diffusion to advection over the characteristic length L. A small Peclet number indicates that the transport of a substance is mainly due to diffusion (Liggett, 1994).

(b) Governing Equation

It is necessary for the constituent within an elemental volume to satisfy the law of conservation of mass, i.e.,

Net rate of change of mass of constituent
= efflux of constituent out of the element -
influx of constituent into the element +
loss or gain of constituent due to reactions

We may combine the different transport processes to obtain the following general equation for one-dimensional flow:

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) + RC + S \quad (12.77)$$

In which R is the reaction rate and S is the source term. This equation is called advection-dispersion equation. Note that this form of mass conservation is valid for transport in pipes, open channel, ponds and porous media. The main difference is in the manner in which the dispersion is quantified in each system, along with the fact that partitioning may take place in porous media due to the presence of solid particles.

The dispersion coefficient D for pipe flow may be determined from the following equation (Holly, 1975):

$$D = 10.1 R_o u_* \quad (12.78)$$

where,

R_o = pipe radius

u_* = shear velocity

For dispersion in waterways, the following equation (Holly, 1985) may be used to estimate D:

$$D = 5.93 u_* h \quad (12.79)$$

in which h = flow depth.

In porous media, one speaks of longitudinal and transverse dispersion, i.e.:

$$D_L = \alpha_L V \quad (12.80)$$

$$D_T = \alpha_T V \quad (12.81)$$

in which α_L and α_T are the longitudinal and transverse dispersivities and V is the seepage velocity.

The U.S. Environmental Protection Agency suggests the following expressions (U.S. EPA, 1986):

$$\alpha_L = 0.1 x_r \quad (12.82)$$

$$\alpha_T = 0.33\alpha_L \quad (12.83)$$

where x_r is the transport distance from the source.

12.7.2 Particulate Settling

(a) Shear Drag and Pressure Drag

Flow around submerged objects will develop two basic types of resistance or drag: shear drag and pressure drag. Pure shear drag is developed by the flow around a flat plate or a disk oriented parallel to the flow. Pure pressure drag is developed by the flow around a flat plate or a disk oriented perpendicularly to the flow. In most problems, however, both types of drag occur.

The general drag equation for flow around submerged objects is:

$$F_D = \frac{C_D A \rho V_o^2}{2} \quad (12.84)$$

where F_D is the drag, C_D is a drag coefficient, A is the projected cross-sectional area of the object in the direction of flow, ρ is the density of the fluid, and V_o is the velocity of the ambient fluid.

When the Reynolds number is very small, say, Re < 0.5, the flow about a submerged object is laminar and the shape of the object is of secondary importance in regard to the drag as compared with the size of the object, the viscosity of the fluid, and the velocity of flow. The drag coefficients for various objects in laminar flow are shown in Table 12.2.

For purely laminar flow around a sphere, Stokes developed a theory, which has been proved by experiment to be accurate. The theory involves the following: (1) The shear drag is two-thirds and the pressure is one-third, of the total drag; (2) at all points on a sphere the longitudinal components of shear drag and pressure drag are combined to produce the same value of unit total drag over the entire surface of the sphere; and (3) the total drag on the sphere is equal to the product of the surface area of the sphere and the unit total drag, or:

$$F_D = \pi d^2 \frac{3\mu V_o}{d} = 3\pi d\mu V_o \qquad (12.85)$$

where d is the diameter of the sphere, μ is the dynamic viscosity and V_o is the terminal velocity of the sphere. When combined with Equation 12.84 this equation will produce the drag coefficient for a sphere as listed in Table 12.2.

At small Reynolds numbers the influence of inertia is insignificant compared with the influence of viscosity. As the Reynolds number is increased, the influence of inertia becomes increasingly pronounced, until eventually at large Reynolds numbers, the situation is completely reversed and the influence of viscosity becomes small compared with the influence of inertia. The variations of the drag coefficient with Reynolds number for several submerged objects are shown in Figure 12.22. It can be seen that the change from one condition to another usually takes place gradually. The sudden decrease in C_D near Re = 2×10^5 for rounded objects is caused by a change from a laminar boundary layer to a turbulent boundary layer and by the resulting change in location of the point of separation of flow.

(b) Fall Velocity

Frequently, in the analysis of sediment and other falling bodies, the size and weight of particles are known and it is desirable to determine the velocity of fall in a fluid. Equation 12.85 can be used to determine the fall velocity V_o of a spherical or nearly spherical particle in a fluid since the drag is the weight of the particle minus the buoyancy force.

At high Reynolds numbers, the velocity must be determined by trial and error from the plot of C_D versus Re in Figure 12.22. However, a direct solution can be made by plotting, with either C_D or Re, a parameter, which does not contain the velocity. This has been done in Figure 12.22 by the scale of $F_D/\rho v^2$, which can be obtained by dimensional analysis or by the following relationship:

$$C_D (\mathrm{Re})^2 = \frac{F_D}{A\rho V^2/2}\left(\frac{Vd}{\nu}\right)^2 \sim \frac{F_D}{\rho \nu^2} \qquad (12.86)$$

At any point on the diagonal broken lines for constant values of $F_D/\rho\nu^2$, the corresponding values of C_D and Re are those required to satisfy the particular values of F_D, ρ, and ν. In fact, the $F_D/\rho\nu^2$ scale can be employed to determine the fall velocity of any object by using the submerged weight of the object as F_D and following down the proper $F_D/\rho\nu^2$ line to the curve for the shape of object involved, from which either C_D or Re can be determined to solve for the fall velocity.

Table 12.2 Drag Coefficients for Laminar Flow.

Object	Range of Re	Value of C_D
Sphere	<0.5	24/Re
Disk perpendicular to flow	<0.5	20.4/Re
Disk parallel to flow	<0.1	13.6/Re
Circular cylinder	<0.1	8π/Re (2.0 – lnRe)

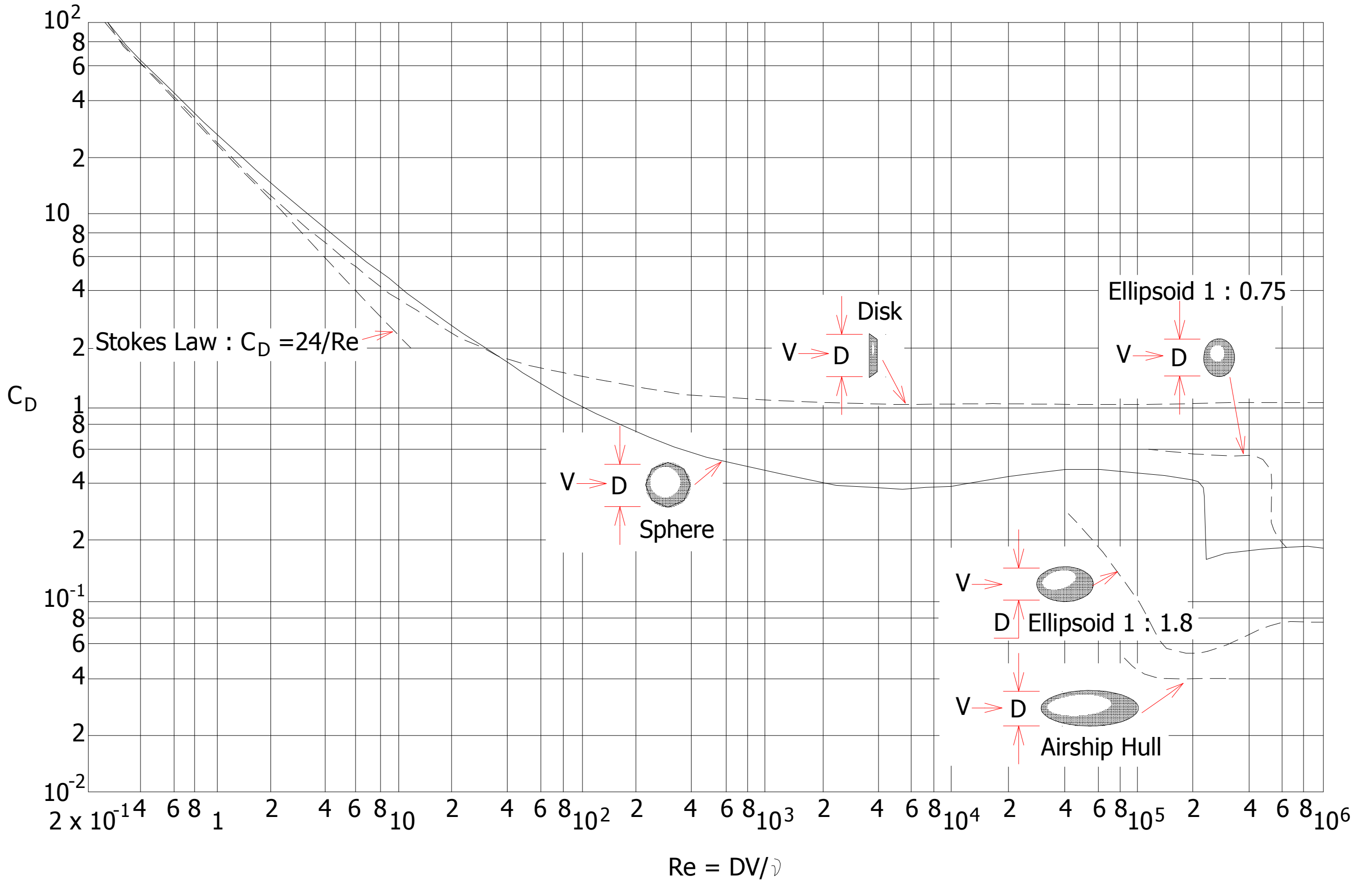


Figure 12.22 Drag Coefficients for spheres and other bodies of revolutions (Prandtl, 1923 and Eisner, 1930).